

PROBABILITY THEORY

Summary:

- 1. Probability theory deals with events of chances.*
- 2. An event is a single result of an experiment*
- 3. A set of all possible outcomes of an experiment is called a sample space or possibility space*
- 4. Probability of an event = $\frac{\text{Number of desired outcomes}}{\text{Number of possible outcomes}}$.*
- 5. Probabilities can be expressed as fractions, decimals or percentages*
- 6. (i) The probability of an event lies between 0 and 1*
(ii) The probability of an impossible event is 0
(iii) The probability of a sure event is 1
- 7. (i) The probability of event A occurring is written as $P(A)$.*
(ii) The probability of event A not occurring is written as $P(\bar{A})$ or $P(A')$.
(iii) For any event A, $P(A) + P(\bar{A}) = 1$ or $P(\bar{A}) = 1 - P(A)$

EXAMPLES:

- 1. An integer between 1 and 10 inclusive is chosen at random. Find the probability that the chosen integer is:*
 - (i) more than 4*
 - (ii) an even number*
 - (iii) a multiple of 3*

(iv) a triangular number

2. *A box contains 3 red, 5 green and 12 blue pens. Find the probability that the pen picked from the box is:*

(i) red

(ii) green

(iii) blue

(iv) not green

(v) not blue

(vi) either red or blue

3. *A box contains red, green and blue pens. The probability of picking a blue pen from the box is $\frac{4}{9}$ and that of a green pen is $\frac{1}{3}$. Find the probability of picking a red pen*

4. *A box contains 8 red pens and the rest are blue. The probability of picking a blue pen from the box is $\frac{3}{5}$. Find the number of blue pens in the box*

5. *A fair coin is thrown once.*

(a) Write down its sample space

(b) Find the probability of obtaining:

(i) a head

(ii) a tail

6. *Two fair coins are tossed together.*

(a) Write down its sample space

(b) Find the probability of obtaining:

(i) two heads

(ii) a head and a tail

7. Three fair coins are tossed simultaneously.

(a) Write down its sample space

(b) Find the probability of obtaining:

(i) exactly two heads

(ii) at least two heads

8. A fair dice is thrown once.

(a) Write down its sample space

(b) Find the probability of obtaining:

(i) an even number

(ii) a square number

(iii) either a 3 or a 4

9. Two fair dice are thrown together.

(a) Draw a table for the possible outcomes

(b) Find the probability that:

(i) both show even numbers

(ii) both show similar faces

(iii) one shows an even number and the other odd

(iv) the sum of the scores is 9

10. Two fair dice are thrown simultaneously. Find the probability that the sum of the scores is

(i) five

(ii) less than five

(iii) greater than six

11. *A fair coin and a fair die are thrown together. Find the probability of obtaining a tail and an even number*

12. *Two fair coins and a fair die are thrown together. Find the probability of obtaining:*

(i) two heads and a number greater than 3

(ii) at least one tail and a number less than 4

13. *A two digit number is formed using the digits 2, 3 and 4 without repeating any digit in the number formed.*

(a) Write down the possibility space for the numbers formed

(b) Find the probability that the number formed is:

(i) even

(ii) less than 30

14. *A two digit number is formed using the digits 0, 3 and 5 without repeating any digit in the number formed.*

(i) Write down the possibility space for the numbers formed

(ii) Find the probability that the number formed is less than 53

Soln:

Hint: *03 and 05 are non–two digit numbers since 03 is the same as 3 and 05 is 5*

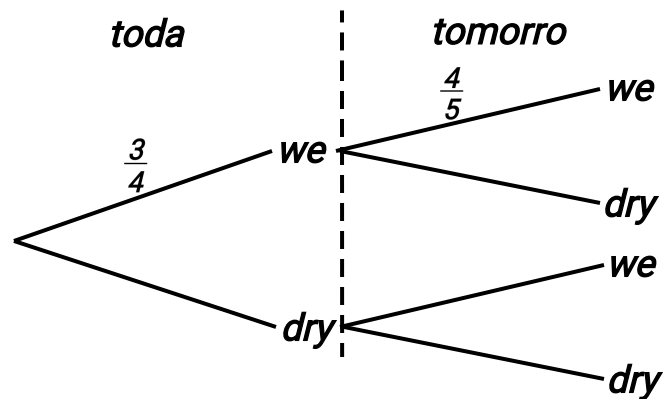
15. *A three digit number is formed using the digits 3, 4 and 5 without repeating any digit in the number formed.*

(i) Write down the possibility space for the numbers formed

(ii) Find the probability that the number formed is even

16. The weather forecast says the probability that it will be wet today is $\frac{3}{4}$ and that of tomorrow is $\frac{4}{5}$.

(i) Copy and complete the probability tree diagram below:



(ii) Find the probability that it will be wet on just one of the two days

17. A coin is biased so that the probability of tossing a head is $\frac{2}{3}$. If it is tossed three times,

(a) Draw a tree diagram showing the possible outcomes

(b) Find the probability of obtaining:

(i) exactly two heads

(ii) at least two tails

(iii) two tails on the first two tosses

18. A box contains 5 red and 3 blue pens. Two pens are drawn in succession at

random from the box with replacement.

(a) Draw a tree diagram showing the possible outcomes

(b) Find the probability of picking:

(i) pens of the same colour

(ii) exactly one red pen

(iii) at least one red pen

(iv) at most one blue pen

19. A box contains **8** red and **3** blue pens. Two pens are drawn in succession at

random from the box without replacement.

(a) Draw a tree diagram showing the possible outcomes

(b) Find the probability of picking:

(i) pens of different colours

(ii) exactly one red pen

(iii) at least one red pen

(iv) at most one blue pen

20. A bag contains **5** red, **7** blue and **8** green beads. Three beads are drawn in succession at random from the bag without replacement.

(a) Draw a tree diagram showing the possible outcomes

(b) Find the probability of picking:

(i) three green beads

(ii) one bead of each colour

(iii) beads of the same colour

21. A bag contains **30** white, **20** blue and **20** red balls. Three balls are drawn in

succession at random without replacement. Find the probability that the first

ball and third ball is white.

EER:

1. An integer between **10** and **30** inclusive is chosen at random. Find the probability that the chosen integer is:

(i) prime

(ii) divisible by **2, 3** or **5**

(iii) a triangular number

(iv) a factor of **240**

2. A letter is chosen from the word **"MISCELLANEOUS"**. Find the probability that it will be:

(i) an **S**

(ii) a vowel

(iii) a consonant

3. In a game, a player tosses a fair coin once. When a **1** or **6** appears the player wins and when a **3** or **4** appears the player losses. Find the probability that the player neither wins nor losses.

4. A two digit number is formed using the digits **1, 2, 4, 5** and **7** without repeating any digit in the number formed.

(a) Write down the possibility space for the numbers formed

(b) Find the probability that the number formed is:

(i) divisible by **5**

(ii) divisible by **2** or **5**

(iii) greater than **50**

5. Two fair dice are thrown together. Find the probability that the product of the scores is

(i) twelve

(ii) four

6. *A fair coin is tossed three times.*

(a) Draw a tree diagram showing the possible outcomes

(b) Find the probability of obtaining:

(i) three heads

(ii) exactly two heads

(iii) at least two tails

7. *Two fair dice are thrown together. The score is the positive difference of the two numbers on which the dice land.*

(a) Construct a table showing the possible outcomes

(b) Find the probability that the positive difference of the scores is

(i) two

(ii) five

8. *A box contains 15 red pens and the rest are blue. The probability of picking a red pen from the box is $\frac{3}{5}$. Find the:*

(i) probability of picking a blue pen

(ii) number of pens in the box

9. *A bag contains 3 white and 5 black balls. If two balls are picked in succession*

at random from it, find the probability that the second ball picked is white if

picking is done:

(i) with replacement.

(ii) without replacement.

10. *Three balls are drawn at random one after the other without replacement from*

a bag containing 4 white, 8 blue, 5 red and 3 pink balls. Find the probability

that the first ball is blue, the second red or blue and the third is white.

11. *A box contains two types of balls **red** and **black**. When a ball is picked*

from the box, the probability that it is red is $\frac{7}{12}$. Two balls are selected at

random from the box without replacement. Find the probability that:

the second ball is black

12. *A box contains 3 red, 4 green, and 5 blue beads. Two beads are selected at*

random from the box without replacement. Find the probability of picking:

(i) beads of the same colour

(ii) at least one red bead

(iii) beads of different colours

13. *A fair die and a fair octahedral die are thrown together and the outcome on each die is recorded.*

(a) Draw a table showing the possible outcomes

(b) Find the probability that:

(i) both show odd numbers

(ii) both show similar faces

(iii) one shows an even number and the other odd

(iv) the sum of the scores is greater than or equal to 5

(v) the product of the scores is 4 or 12

14. A bag contains x red balls and $(x - 8)$ blue ones. The probability of picking a red ball from the box is 0.75. Find the:

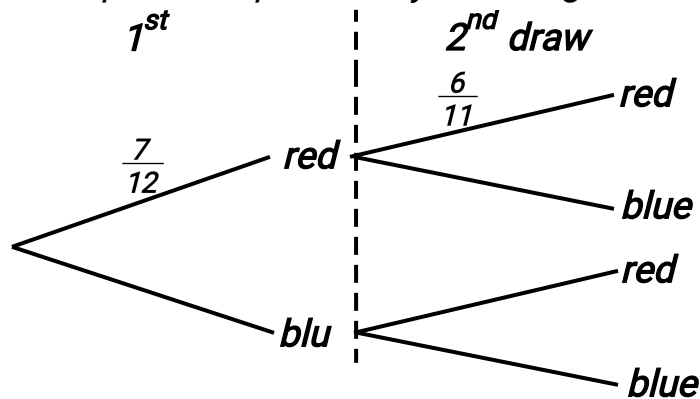
(i) probability of picking a blue ball

(ii) number of balls in the bag

15. A box contains 7 red and 5 blue pens. Two pens are drawn in succession at

random from the box without replacement.

(i) Copy and complete the probability tree diagram below:



(ii) Find the probability of picking at least one red pen

16. A fair die and a fair tetrahedral die are thrown together and the outcome on each die is recorded.

(a) Draw a table showing the possible outcomes

(b) Find the probability that:

(i) both show odd numbers

(ii) the sum of the scores is greater than or equal to 5

(iii) the product of the scores is 4 or 12

17. Two fair dice are thrown together.

(a) Draw a table for the possible outcomes

(b) Find the probability that:

(i) both show even numbers

(ii) both show similar faces

(iii) one shows an even number and the other odd

(iv) the sum of the scores is 9

18. *Two fair tetrahedral dice each with faces numbered 1 to 4 are thrown together. The score is the sum of the two numbers on which the dice land.*

(a) Construct a table showing the possible outcomes

(b) Find the probability that the sum of the scores is

(i) seven

(ii) less than six

19. *Two fair dice are thrown together. Find the probability that the positive difference of the scores is*

(i) two

(ii) five

20. *A box contains 5 red and 3 blue balls. Two balls are drawn in succession at*

random from it without replacement. Find the probability that:

(i) they are of the same colour

(ii) the second ball drawn is red.

21. *A box contains 5 red and 3 blue balls. Three balls are selected in succession at*

random from it without replacement. Find the probability that:

(i) they are of the same colour

(ii) the first and last are of the same colour

(iii) at most one blue ball is drawn.

22. *Two fair tetrahedral dice are thrown up at once. Find the probability that the sum of the scores on the dice is less than 6.*